Presentation 1

$\tilde{y}_n$

$$H(y) = \langle T_1, T_2, \dots, T_{n-1}, T_0, \pi \rangle \qquad \left.\begin{array}{ll} T_iT_j = T_jT_i & |i-j|>1 \\ T_iT_{i+1}T_i = T_{i+1}T_iT_{i+1} & \end{array}\right\} \text{braid "mod } n\text{"}$$

$$(T_i - t)(T_i + 1) = 0 \quad \text{quadratic} \qquad \pi T_i \pi^{-1} = T_{i+1}$$

Action: $T_i f = t f^{s_i} + (t-1)\dfrac{f - f^{s_i}}{1 - X^{\alpha_i}}$ $\qquad \pi f(x_1, \dots, x_n) = f(x_2, \dots, x_n, q^{-1}x_1)$

where $s_i : x_i \leftrightarrow x_{i+1}$ $\qquad s_0 : x_1 \leftrightarrow q x_n$ $\qquad X^{\alpha_i} = \begin{cases} x_i / x_{i+1} & i \neq 0 \\ q x_n / x_1 & i = 0 \end{cases}$

Recall $q = X^{\delta}$

Presentation 2

$S_n \ltimes P$

$$H(y) = \langle T_1, \dots, T_{n-1}, y_1^{\pm 1}, \dots, y_n^{\pm 1} \rangle \qquad y_iy_j = y_jy_i \qquad T_i y_i^{-1} T_i = t y_{i+1}^{-1} \qquad T_i y_j = y_j T_i \quad j \neq i, i+1$$

$$y_1 = T_1 T_2 \cdots T_{n-1} \pi^{-1}$$

Action: blech

R = reduced root system; $\hat{R}$ = affine root system; δ = null root

nonsymmetric	symmetric
$\Delta^R = \Delta^R_{q,t} = \prod_{\alpha\in\hat{R}_+} \frac{1-x^\alpha}{1-\underline{t}x^\alpha}$	$\tilde{\Delta}^R = \tilde{\Delta}^R_{q,t} = \Delta^R_{q,t}\,\overline{\Delta^R_{q,t}} = \prod_{\alpha\in\hat{R}} \frac{1-x^\alpha}{1-tx^\alpha}$

Let $q = x^\delta$

$$\Delta^A_{q,t} = \prod_{1\le i<j\le n}\prod_{k=0}^{\infty} \frac{\left(1-q^k\frac{x_i}{x_j}\right)\left(1-q^{k+1}\frac{x_j}{x_i}\right)}{\left(1-\underline{t}q^k\frac{x_i}{x_j}\right)\left(1-\underline{t}q^{k+1}\frac{x_j}{x_i}\right)}$$

$$\Delta = \Delta^C_{q,t} = \prod_{1\le i\le n}\prod_{k=0}^{\infty} \frac{(1-q^kz_i^2)(1-q^{k+1}z_i^{-2})}{(1-tq^kz_i^2)(1-tq^{k+1}z_i^{-2})} \prod_{1\le i<j\le n}\prod_{k=0}^{\infty} \frac{(1-q^kz_iz_j^{\pm1})(1-q^{k+1}z_i^{-1}z_j^{\pm1})}{(1-tq^kz_iz_j^{\pm1})(1-tq^{k+1}z_i^{-1}z_j^{\pm1})}$$

Note: $\Delta^C(z_1,\dots,z_n) = \Delta^A(z_1,\dots,z_n,z_n^{-1},\dots,z_1^{-1})$

Also $\Delta^C = \Delta^K(a,b,c,d;q,t)$ Koornwinder $\quad a=-b=t^{1/2}$, $c=-d=(qt)^{1/2}$